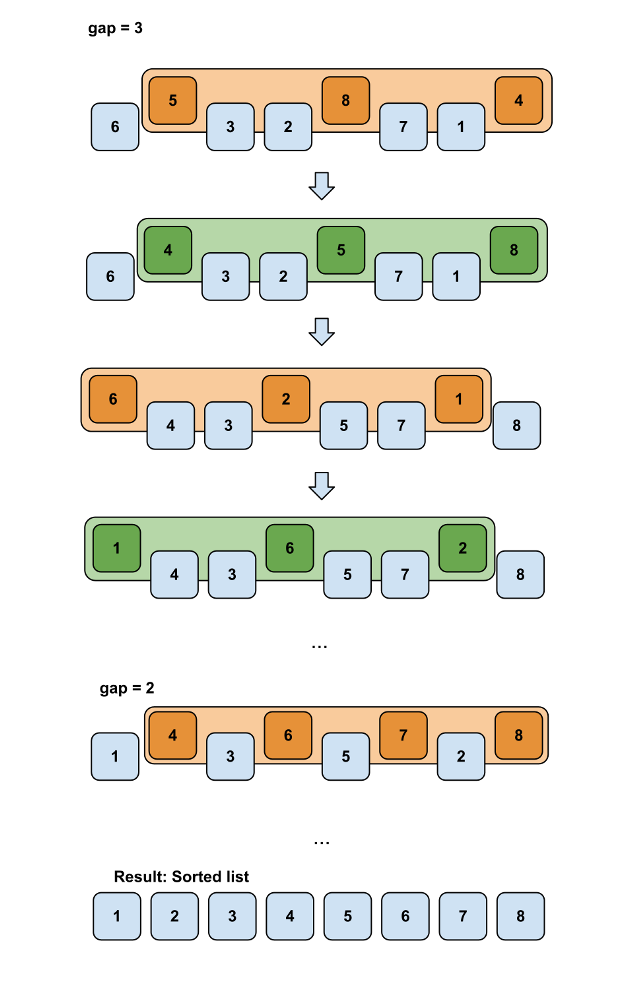
How Will Different Gap Sequences Affect the Performance of the Shellsort Algorithm?

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How will different gap sequences affect the performance of the Shellsort algorithm? There are many different types of algorithms used in computer applications to sort collections of data in various ways. Shellsort is one such algorithm. Invented by Donald Shell in 1959, Shellsort takes an interesting approach to sorting. A so-called gap sequence is used by the algorithm to iterate through the given data, sorting it in place. The most fascinating feature of this method, is the fact that the time it takes to sort the given data varies greatly depending on the gap sequence used. With my science fair, I aim to tackle the question of what sets of numbers are most efficient when used as gap sequences to sort an arbitrary set of integers.



Visualizing Shellsort

A very important notion to consider when choosing or designing a sorting algorithm is asymptotic computational complexity. This concept describes the worst-case complexity of an algorithm and can help estimate the amount of time it will take to perform. Shellsort is also interesting in this respect, as its worse-case time complexity depends on the gap sequence used! This science fair is also quite unique because the best gap sequence to use for Shellsort is currently unknown.

The independent variable for my science fair is the set of unique, positive integers used as a gap sequence for the Shellsort algorithm. This is defined by the user, namely myself. An example of such a sequence is \lfloor N / 2^k \rfloor. For N = 1000, the sequence would become: [500, 250, 125, 62, 31, 15, 7, 3, 1]. The dependant variable is the amount of time, in milliseconds, it takes the algorithm to perform a sort on an array of N integers. This will be timed by interacting with the computer’s system clock using Java. The program will be run on numerous values of N using different gap sequences. Control variables include the computer on which the program is run and the implementation of the Shellsort algorithm. My hypothesis is that as N (the size of the array to be sorted) increases, Shellsort will become more efficient when a more effective gap sequence is used because Shellsort allows the exchanging of far apart elements, whereas more rudimentary sorts like insertion sort or bubble sort only swap adjacent elements.

**Materials**

* Computer with a JRE (Java Runtime Environment) installed
* An IDE (integrated development environment) on which to develop the application

**Procedure**

1. Shellsort was implemented using Java in the Eclipse IDE in just under 300 lines of code
2. 7 suitable gap sequences with which to run Shellsort were identified
3. The aforementioned gap sequences were entered into the code
4. The given sequences were used to run Shellsort on different values of N (different array sizes)
5. The time taken to sort an array of N integers were averaged out over 10 trials for every value of N using each sequence
6. These times were written to a comma-separated values (CSV) file
7. Times in the CSV file were graphed using Excel
8. The most effective gap sequence was determined by the time it took to run (faster is better)

**Raw Data**

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383000.0,56.79999923706055,51.099998474121094,53.099998474121094,44.900001525878906,31.100000381469727,51.099998474121094,41.0

384000.0,58.29999923706055,49.29999923706055,53.400001525878906,45.29999923706055,30.899999618530273,56.400001525878906,41.5

385000.0,57.400001525878906,51.599998474121094,53.5,47.0,31.100000381469727,51.099998474121094,41.099998474121094

386000.0,57.5,50.099998474121094,53.70000076293945,45.29999923706055,31.299999237060547,50.29999923706055,41.20000076293945

387000.0,59.400001525878906,52.400001525878906,53.79999923706055,46.099998474121094,31.600000381469727,50.29999923706055,41.20000076293945

388000.0,66.0,51.900001525878906,54.099998474121094,45.70000076293945,31.5,51.5,41.400001525878906

389000.0,60.900001525878906,50.70000076293945,54.5,45.5,31.799999237060547,51.70000076293945,41.79999923706055

390000.0,58.29999923706055,50.29999923706055,53.599998474121094,46.099998474121094,31.700000762939453,51.29999923706055,41.900001525878906

391000.0,58.5,52.599998474121094,53.900001525878906,46.400001525878906,31.5,51.599998474121094,41.70000076293945

392000.0,59.20000076293945,52.70000076293945,54.29999923706055,46.20000076293945,32.0,52.79999923706055,41.70000076293945

393000.0,59.400001525878906,53.0,55.20000076293945,47.20000076293945,32.0,53.0,41.599998474121094

394000.0,59.20000076293945,51.29999923706055,54.70000076293945,47.0,32.099998474121094,57.79999923706055,41.79999923706055

395000.0,59.099998474121094,52.0,54.5,47.20000076293945,32.5,55.29999923706055,42.20000076293945

396000.0,60.099998474121094,52.900001525878906,55.20000076293945,46.79999923706055,32.29999923706055,53.29999923706055,42.29999923706055

397000.0,61.099998474121094,51.70000076293945,55.400001525878906,47.5,32.5,53.400001525878906,42.5

398000.0,65.0,52.29999923706055,55.599998474121094,47.20000076293945,32.599998474121094,51.70000076293945,42.70000076293945

399000.0,60.0,52.79999923706055,55.70000076293945,47.20000076293945,32.599998474121094,52.400001525878906,42.79999923706055

400000.0,61.20000076293945,52.20000076293945,56.0,47.29999923706055,32.5,54.70000076293945,42.79999923706055

401000.0,60.400001525878906,51.79999923706055,56.0,48.29999923706055,32.70000076293945,53.29999923706055,43.0

402000.0,60.79999923706055,52.900001525878906,56.29999923706055,47.70000076293945,32.599998474121094,52.400001525878906,43.0

403000.0,61.400001525878906,51.79999923706055,56.29999923706055,48.29999923706055,32.70000076293945,53.79999923706055,44.29999923706055

404000.0,61.599998474121094,53.900001525878906,56.400001525878906,48.20000076293945,32.900001525878906,54.0,44.70000076293945

405000.0,62.0,52.599998474121094,56.400001525878906,48.400001525878906,32.900001525878906,54.29999923706055,44.70000076293945

406000.0,65.30000305175781,52.0,57.29999923706055,48.29999923706055,33.0,53.20000076293945,45.599998474121094

407000.0,66.80000305175781,54.0,57.0,48.20000076293945,33.20000076293945,52.70000076293945,43.5

408000.0,62.099998474121094,51.599998474121094,57.0,48.70000076293945,33.0,54.79999923706055,43.79999923706055

409000.0,62.599998474121094,53.599998474121094,57.099998474121094,48.900001525878906,33.400001525878906,54.599998474121094,43.79999923706055

410000.0,64.5999984741211,54.099998474121094,57.29999923706055,49.5,33.400001525878906,55.5,43.900001525878906

411000.0,63.29999923706055,54.29999923706055,57.5,49.20000076293945,33.29999923706055,53.70000076293945,44.599998474121094

412000.0,64.9000015258789,54.900001525878906,58.29999923706055,49.5,33.70000076293945,53.5,48.0

413000.0,66.0,54.099998474121094,58.400001525878906,49.599998474121094,33.79999923706055,53.70000076293945,45.900001525878906

414000.0,69.9000015258789,54.5,57.79999923706055,49.5,33.5,53.5,44.5

415000.0,64.5,56.099998474121094,58.599998474121094,49.900001525878906,33.900001525878906,53.400001525878906,45.29999923706055

416000.0,64.5999984741211,54.400001525878906,58.79999923706055,49.79999923706055,34.099998474121094,57.0,44.5

417000.0,72.80000305175781,55.20000076293945,58.5,50.099998474121094,33.900001525878906,54.5,45.0

418000.0,65.19999694824219,54.29999923706055,58.70000076293945,50.400001525878906,34.099998474121094,55.599998474121094,44.5

419000.0,65.30000305175781,54.099998474121094,59.29999923706055,49.79999923706055,34.20000076293945,54.20000076293945,44.79999923706055

420000.0,65.19999694824219,55.29999923706055,58.70000076293945,50.599998474121094,34.20000076293945,55.29999923706055,45.20000076293945

421000.0,66.0,54.5,59.599998474121094,50.599998474121094,34.29999923706055,55.400001525878906,45.29999923706055

422000.0,65.5999984741211,53.79999923706055,58.70000076293945,51.29999923706055,34.5,56.900001525878906,45.0

423000.0,66.0999984741211,56.29999923706055,59.70000076293945,51.0,34.20000076293945,55.20000076293945,46.20000076293945

424000.0,66.5,54.5,59.599998474121094,50.79999923706055,34.70000076293945,56.79999923706055,47.0

425000.0,68.5,56.29999923706055,60.099998474121094,51.400001525878906,34.900001525878906,56.20000076293945,46.0

426000.0,73.0,56.79999923706055,60.20000076293945,51.0,34.599998474121094,83.9000015258789,52.900001525878906

427000.0,68.19999694824219,55.900001525878906,60.099998474121094,51.5,35.0,58.099998474121094,54.79999923706055

428000.0,68.69999694824219,56.0,60.400001525878906,51.70000076293945,35.20000076293945,58.0,54.599998474121094

429000.0,67.5,57.0,60.29999923706055,52.900001525878906,35.099998474121094,56.5,48.400001525878906

430000.0,67.5999984741211,56.79999923706055,61.099998474121094,51.400001525878906,35.20000076293945,56.70000076293945,46.70000076293945

431000.0,68.19999694824219,56.400001525878906,60.29999923706055,51.70000076293945,35.099998474121094,55.400001525878906,46.29999923706055

432000.0,68.19999694824219,56.79999923706055,60.599998474121094,51.5,35.400001525878906,58.29999923706055,46.29999923706055

433000.0,67.9000015258789,57.29999923706055,60.599998474121094,52.79999923706055,35.599998474121094,56.900001525878906,46.79999923706055

434000.0,68.19999694824219,57.70000076293945,61.20000076293945,52.5,35.5,57.599998474121094,47.099998474121094

435000.0,69.19999694824219,57.400001525878906,61.70000076293945,52.5,35.599998474121094,56.0,46.5

436000.0,69.5,57.20000076293945,61.599998474121094,52.099998474121094,36.0,59.0,47.20000076293945

437000.0,69.5999984741211,59.29999923706055,62.20000076293945,53.20000076293945,35.79999923706055,57.79999923706055,47.5

438000.0,70.0,60.0,62.099998474121094,53.0,35.599998474121094,58.29999923706055,47.400001525878906

439000.0,70.0,61.0,62.400001525878906,52.79999923706055,36.099998474121094,60.900001525878906,48.099998474121094

440000.0,70.0999984741211,60.0,62.599998474121094,52.70000076293945,36.20000076293945,60.20000076293945,47.70000076293945

441000.0,70.30000305175781,60.900001525878906,61.900001525878906,53.400001525878906,36.400001525878906,58.400001525878906,49.20000076293945

442000.0,72.0999984741211,61.5,62.0,53.20000076293945,36.20000076293945,61.0,48.29999923706055

443000.0,71.69999694824219,57.79999923706055,62.599998474121094,53.599998474121094,36.400001525878906,61.099998474121094,48.400001525878906

444000.0,73.80000305175781,57.900001525878906,63.099998474121094,53.5,36.400001525878906,59.0,47.70000076293945

445000.0,79.0,59.70000076293945,63.599998474121094,53.70000076293945,36.599998474121094,57.79999923706055,48.20000076293945

446000.0,79.0999984741211,57.599998474121094,62.900001525878906,53.70000076293945,36.900001525878906,59.0,47.79999923706055

447000.0,72.0999984741211,60.400001525878906,63.099998474121094,54.0,36.900001525878906,58.0,48.29999923706055

448000.0,72.5,59.79999923706055,63.900001525878906,54.0,36.70000076293945,63.70000076293945,48.400001525878906

449000.0,72.0,59.5,63.79999923706055,54.29999923706055,37.0,59.400001525878906,48.400001525878906

450000.0,72.0,61.20000076293945,64.5999984741211,54.400001525878906,37.20000076293945,60.599998474121094,48.70000076293945

451000.0,72.19999694824219,58.79999923706055,64.0,54.70000076293945,37.0,59.70000076293945,49.0

452000.0,73.30000305175781,59.70000076293945,64.5,55.099998474121094,37.20000076293945,59.79999923706055,48.79999923706055

453000.0,73.5,60.099998474121094,64.30000305175781,54.5,37.5,59.79999923706055,48.79999923706055

454000.0,73.0999984741211,61.5,64.5999984741211,55.099998474121094,37.20000076293945,60.599998474121094,49.599998474121094

455000.0,73.5,60.29999923706055,65.4000015258789,55.400001525878906,37.900001525878906,59.79999923706055,51.599998474121094

456000.0,74.0999984741211,63.79999923706055,64.69999694824219,55.400001525878906,37.599998474121094,61.79999923706055,50.599998474121094

457000.0,74.30000305175781,60.900001525878906,64.80000305175781,55.599998474121094,38.0,61.400001525878906,49.400001525878906

458000.0,74.4000015258789,61.900001525878906,65.4000015258789,55.70000076293945,38.0,65.9000015258789,49.70000076293945

459000.0,75.0,60.79999923706055,65.19999694824219,55.79999923706055,37.900001525878906,78.30000305175781,49.599998474121094

460000.0,74.5999984741211,61.0,64.80000305175781,56.099998474121094,38.20000076293945,64.69999694824219,49.5

461000.0,75.0,61.099998474121094,65.9000015258789,55.599998474121094,38.20000076293945,61.70000076293945,50.099998474121094

462000.0,75.30000305175781,60.599998474121094,65.0999984741211,55.599998474121094,38.599998474121094,62.5,50.099998474121094

463000.0,75.5,61.0,66.0999984741211,56.400001525878906,38.099998474121094,60.900001525878906,50.099998474121094

464000.0,75.69999694824219,61.5,65.5,56.900001525878906,38.70000076293945,63.20000076293945,50.29999923706055

465000.0,76.19999694824219,61.0,66.4000015258789,56.70000076293945,38.400001525878906,61.400001525878906,50.5

466000.0,76.5999984741211,60.79999923706055,67.30000305175781,56.5,38.70000076293945,62.5,50.29999923706055

467000.0,76.5,61.29999923706055,66.5999984741211,57.099998474121094,38.900001525878906,66.19999694824219,50.70000076293945

468000.0,77.80000305175781,62.5,66.69999694824219,57.400001525878906,39.0,61.79999923706055,51.0

469000.0,76.5999984741211,62.400001525878906,67.0,57.400001525878906,38.900001525878906,66.19999694824219,50.79999923706055

470000.0,77.5999984741211,64.30000305175781,67.30000305175781,57.20000076293945,39.099998474121094,63.29999923706055,51.29999923706055

471000.0,77.9000015258789,64.80000305175781,67.30000305175781,57.20000076293945,38.900001525878906,62.900001525878906,51.099998474121094

472000.0,77.5,62.29999923706055,67.5999984741211,57.20000076293945,39.29999923706055,64.0,51.099998474121094

473000.0,78.4000015258789,63.5,67.69999694824219,57.70000076293945,39.099998474121094,63.20000076293945,50.900001525878906

474000.0,78.69999694824219,62.400001525878906,68.4000015258789,57.5,39.20000076293945,64.5,51.0

475000.0,78.19999694824219,63.599998474121094,68.19999694824219,57.70000076293945,39.5,64.80000305175781,51.0

476000.0,78.5999984741211,64.0,68.80000305175781,57.900001525878906,39.400001525878906,68.19999694824219,51.400001525878906

477000.0,79.19999694824219,64.19999694824219,68.5999984741211,58.20000076293945,40.29999923706055,65.5,51.599998474121094

478000.0,79.0,64.19999694824219,68.80000305175781,58.400001525878906,39.79999923706055,63.29999923706055,51.5

479000.0,79.5,63.400001525878906,68.5999984741211,60.0,40.0,64.19999694824219,51.599998474121094

480000.0,80.4000015258789,66.0,68.69999694824219,59.70000076293945,40.0,66.80000305175781,52.20000076293945

481000.0,80.5,67.4000015258789,69.4000015258789,59.0,40.099998474121094,64.0,51.79999923706055

482000.0,80.69999694824219,66.30000305175781,69.30000305175781,59.20000076293945,40.099998474121094,65.0,52.20000076293945

483000.0,80.5999984741211,66.0,69.30000305175781,59.20000076293945,40.099998474121094,65.0999984741211,52.400001525878906

484000.0,80.80000305175781,64.30000305175781,70.0999984741211,59.29999923706055,40.20000076293945,64.80000305175781,52.5

485000.0,80.9000015258789,64.5999984741211,70.4000015258789,59.099998474121094,40.400001525878906,64.80000305175781,52.29999923706055

486000.0,81.69999694824219,66.0,69.9000015258789,59.20000076293945,40.599998474121094,65.5999984741211,52.599998474121094

487000.0,81.0999984741211,65.80000305175781,70.19999694824219,59.29999923706055,40.70000076293945,65.5999984741211,52.599998474121094

488000.0,82.19999694824219,65.19999694824219,70.5,59.599998474121094,40.900001525878906,66.69999694824219,52.900001525878906

489000.0,82.80000305175781,66.0999984741211,70.4000015258789,59.5,40.70000076293945,66.19999694824219,52.79999923706055

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491000.0,89.30000305175781,68.0999984741211,70.30000305175781,60.79999923706055,41.0,70.5,53.20000076293945

492000.0,83.19999694824219,65.5999984741211,71.0999984741211,60.400001525878906,40.79999923706055,71.5999984741211,53.5

493000.0,83.9000015258789,66.30000305175781,70.5,59.599998474121094,41.20000076293945,67.19999694824219,53.20000076293945

494000.0,83.5,65.9000015258789,71.19999694824219,60.5,41.20000076293945,66.4000015258789,53.599998474121094

495000.0,83.69999694824219,65.5999984741211,71.5999984741211,60.70000076293945,41.29999923706055,66.69999694824219,53.79999923706055

496000.0,83.69999694824219,65.30000305175781,71.19999694824219,60.79999923706055,41.20000076293945,68.19999694824219,53.599998474121094

497000.0,84.5999984741211,68.80000305175781,71.80000305175781,61.0,41.20000076293945,65.80000305175781,54.0

498000.0,84.80000305175781,65.5999984741211,71.5999984741211,61.20000076293945,41.20000076293945,66.5,54.099998474121094

499000.0,84.69999694824219,66.69999694824219,72.0999984741211,61.0,42.0,67.0999984741211,54.29999923706055

500000.0,84.69999694824219,67.5,72.5,61.70000076293945,41.599998474121094,67.9000015258789,54.400001525878906

Main.java

**package** com.stakoun.shellsortanalysis;

**import** java.io.FileWriter;

**import** java.io.IOException;

**import** java.util.Random;

**public** **class** Main

{

**public** **static** **enum** Sequence {

*CIURA*,

*FRANK\_LAZARUS*,

*HIBBARD*,

*KNUTH*,

*SEDGEWICK*,

*SHELL*,

*TOKUDA*

}

**private** **static** **final** String *file* = "results.csv";

**private** **static** **final** **float** *trials* = 10f;

**private** **static** **final** **int** *maxN* = 500000;

**private** **static** **final** **int** *inc* = 1000;

**private** **static** **final** Sequence[] *sequences* = Sequence.*values*();

**private** **static** **final** **int** *numSequences* = *sequences*.length;

**private** **static** FileWriter *writer*;

**private** **static** Sorter *sorter*;

**private** **static** SequenceGenerator *sequenceGenerator*;

**private** **static** Random *random*;

**private** **static** Sequence *sequence*;

**private** **static** **double**[][] *times*;

**public** **static** **void** main(String[] args) **throws** IOException

{

*writer* = **new** FileWriter(*file*);

*sorter* = **new** Sorter();

*sequenceGenerator* = **new** SequenceGenerator();

*random* = **new** Random();

*times* = **new** **double**[*maxN*/*inc*+1][*numSequences*+1];

**for** (**int** i = 0; i <= *maxN*/*inc*; i++)

*times*[i][0] = i\**inc*;

**long** start = System.*currentTimeMillis*();

**for** (**int** i = 1; i <= *numSequences*; i++) {

*sequence* = *sequences*[i-1];

**long** seqStart = System.*currentTimeMillis*();

**for** (**int** N = 0; N <= *maxN*; N += *inc*) {

*sorter*.setSequence(*sequenceGenerator*.genSequence(*sequence*, N));

**long** total = 0;

**for** (**int** t = 0; t < *trials*; t++)

total += *sorter*.sort(*getRandomArray*(N));

**double** avg = total / *trials*;

*times*[N/*inc*][i] = avg;

}

System.*out*.println(*sequence*.name()+" completed in "+(System.*currentTimeMillis*()-seqStart)+" ms");

}

System.*out*.println("Completed in "+(System.*currentTimeMillis*()-start)+" ms");

**for** (**int** i = 0; i <= *maxN*/*inc*; i++) {

**for** (**int** j = 0; j <= *numSequences*; j++)

*writer*.append(*times*[i][j]+(j!=*numSequences*?",":""));

*writer*.append("\n");

}

*writer*.flush();

*writer*.close();

}

**public** **static** **int**[] getRandomArray(**int** N)

{

**int**[] array = **new** **int**[N];

**for** (**int** i = 0; i < N; i++)

array[i] = *getRandom*(N);

**return** array;

}

**public** **static** **int** getRandom(**int** N) {

**return** *random*.nextInt(N);

}

}

SequenceGenerator.java

**package** com.stakoun.shellsortanalysis;

**import** com.stakoun.shellsortanalysis.Main.Sequence;

**public** **class** SequenceGenerator

{

**public** **int**[] genSequence(Sequence sequence, **int** N)

{

**switch** (sequence)

{

**case** *CIURA*: **return** genCiuraSequence(N);

**case** *FRANK\_LAZARUS*: **return** genFrankLazarusSequence(N);

**case** *HIBBARD*: **return** genHibbardSequence(N);

**case** *KNUTH*: **return** genKnuthSequence(N);

**case** *SEDGEWICK*: **return** genSedgewickSequence(N);

**case** *SHELL*: **return** genShellSequence(N);

**case** *TOKUDA*: **return** genTokudaSequence(N);

**default**: **return** **new** **int**[0];

}

}

**public** **int**[] genCiuraSequence(**int** N)

{

**int**[] ciura = **new** **int**[] {1, 4, 10, 23, 57, 132, 301, 701, 1750};

// Get length of sequence

**int** gap;

**int** k = 0;

**do** {

gap = ciura[k++];

} **while** (k < ciura.length && ciura[k] < N);

// Create sequence

**int**[] res = **new** **int**[k];

k = 0;

**do** {

gap = ciura[k];

res[k++] = gap;

} **while** (k < ciura.length && ciura[k] < N);

**return** reverse(res);

}

**public** **int**[] genFrankLazarusSequence(**int** N)

{

// Get length of sequence

**int** gap;

**int** k = 1;

**do** {

gap = 2 \* (N / pow(2, k+++1)) + 1;

} **while** (gap > 1);

// Create sequence

**int**[] res = **new** **int**[k-1];

k = 1;

**do** {

gap = 2 \* (N / pow(2, k+1)) + 1;

res[k++-1] = gap;

} **while** (gap > 1);

**return** res;

}

**public** **int**[] genHibbardSequence(**int** N)

{

// Get length of sequence

**int** gap;

**int** k = 1;

**do** {

gap = pow(2, k++) - 1;

} **while** (gap < N/2);

// Create sequence

**int**[] res = **new** **int**[k-1];

k = 1;

**do** {

gap = pow(2, k) - 1;

res[k++-1] = gap;

} **while** (gap < N/2);

**return** reverse(res);

}

**public** **int**[] genKnuthSequence(**int** N)

{

// Get length of sequence

**int** gap;

**int** k = 1;

**do** {

gap = (pow(3, k++) - 1) / 2;

} **while** (gap < N);

// Create sequence

**int**[] res = **new** **int**[k-1];

k = 1;

**do** {

gap = (pow(3, k) - 1) / 2;

res[k++-1] = gap;

} **while** (gap < N);

**return** reverse(res);

}

**public** **int**[] genSedgewickSequence(**int** N)

{

// Get length of sequence

**int** gap;

**int** k = 1;

**do** {

gap = pow(4, k) + 3 \* pow(2, k++-1) + 1;

} **while** (gap < N);

// Create sequence

**int**[] res = **new** **int**[k-1];

k = 1;

**do** {

gap = pow(4, k) + 3 \* pow(2, k-1) + 1;

res[k++-1] = gap;

} **while** (gap < N);

**return** reverse(res);

}

**public** **int**[] genShellSequence(**int** N)

{

// Get length of sequence

**int** gap;

**int** k = 1;

**do** {

gap = N / pow(2, k++);

} **while** (gap > 1);

// Create sequence

**int**[] res = **new** **int**[k-1];

k = 1;

**do** {

gap = N / pow(2, k);

res[k++-1] = gap;

} **while** (gap > 1);

**return** res;

}

**public** **int**[] genTokudaSequence(**int** N)

{

// Get length of sequence

**int** gap;

**int** k = 1;

**do** {

gap = ceil((**double**)(pow(9l, k) - pow(4l, k)) / (**double**)(5 \* pow(4l, k++-1)));

} **while** (gap < N);

// Create sequence

**int**[] res = **new** **int**[k-1];

k = 1;

**do** {

gap = ceil((**double**)(pow(9l, k) - pow(4l, k)) / (**double**)(5 \* pow(4l, k-1)));

res[k++-1] = gap;

} **while** (gap < N);

**return** reverse(res);

}

**public** **int** pow(**int** base, **int** exp)

{

**int** res = 1;

**for** (**int** i = 0; i < exp; i++)

res \*= base;

**return** res;

}

**public** **long** pow(**long** base, **int** exp)

{

**long** res = 1;

**for** (**int** i = 0; i < exp; i++)

res \*= base;

**return** res;

}

**public** **int** ceil(**double** d)

{

**if** (d - (**int**)d > 0.00001)

**return** (**int**)d + 1;

**return** (**int**)d;

}

**public** **int**[] reverse(**int**[] arr)

{

**int** len = arr.length;

**int**[] newArr = **new** **int**[len];

**for** (**int** i = 0; i < len; i++)

newArr[i] = arr[len-i-1];

**return** newArr;

}

}

Sorter.java

**package** com.stakoun.shellsortanalysis;

**public** **class** Sorter

{

**private** **int**[] sequence;

**public** **long** sort(**int**[] array)

{

**long** start = System.*currentTimeMillis*();

**for** (**int** gap : sequence) {

**for** (**int** i = gap; i < array.length; i++) {

**int** val = array[i];

**int** j;

**for** (j = i; j >= gap && array[j - gap] > val; j -= gap)

array[j] = array[j - gap];

array[j] = val;

}

}

**return** System.*currentTimeMillis*() - start;

}

**public** **void** setSequence(**int**[] sequence) {

**this**.sequence = sequence;

}

}

|  |  |  |
| --- | --- | --- |
| Author | General Term | Gap Sequence |
| Ciura | Empirically Derived | 1, 4, 10, 23, 57, 132, 301, 701, 1750 |
| Frank & Lazarus | 2 \lfloor N / 2^{k+1} \rfloor + 1 | 2 \left\lfloor\frac{N}{4}\right\rfloor + 1, \ldots, 3, 1 |
| Hibbard | 2^k - 1 | 1, 3, 7, 15, 31, 63, \ldots |
| Knuth | (3k – 1) / 2 | 1, 4, 14, 40, 121, … |
| Sedgewick | 4^k + 3\cdot2^{k-1} + 1 | 1, 8, 23, 77, 281, \ldots |
| Shell | \lfloor N / 2^k \rfloor | \left\lfloor\frac{N}{2}\right\rfloor,         \left\lfloor\frac{N}{4}\right\rfloor, \ldots, 1 |
| Tokuda | \left\lceil \frac{9^k-4^k}{5\cdot4^{k-1}} \right\rceil | 1, 4, 9, 20, 46, 103, \ldots |

My experiment analysed 7 of the most popular gap sequences used with the Shellsort algorithm. The less time it takes for the algorithm to sort an array of size N using the given sequence, the more efficient the gap sequence is. My experiment concluded that Sedgewick’s 1986 sequence, generated by the equation: ak = 4k-1+3×2k-2+1, for k ≥ 2, which creates the gap sequence 1, 8, 23, 77, …, was the most efficient sequence analysed. Tokuda’s and Knuth’s gap sequences were 2nd and 3rd in efficiency respectively.

The Java application written to measure the time taken for each of these gap sequences did 10 trials for values of N from 0 to 500,000 with increments of 1,000 for each of the 7 tested gap sequences (35,070 trials total). The data was then written to a comma-separated values (CSV) file and used to create graphs. For example, Sedgewick’s sequence took an average of 41.6 milliseconds to sort an array of 500,000 integers, while Shell’s sequence took 67.9 milliseconds to do the same. As the value of N increases, less efficient sequences’ time complexities begin to increase more rapidly, as seen in the provided graphs, supporting the hypothesis that more effective gap sequences lead to faster sort times.

This experiment has many real-world applications, as Shellsort can be implemented with very little code and does not use a computer system’s call stack. An efficient gap sequence, such as Sedgewick’s, can significantly reduce the time taken to sort an array of numbers when used in an industry setting, compared to less efficient sequences such as Hibbard’s. A faster sort time leads to more efficient and user-friendly computer applications.

As this was a carefully crafted computer application, there is little room for human error. To achieve more accurate results, one would simply have to change the number of trials the program runs for each value of N. This program could also be extended to simulate the sorting of much higher values of N using different increments.

**Works Cited**

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